LOOK ACADEMIC PUBLISHERS
OPEN ACCESS

INTERNATIONAL JOURNAL OF ENVIRONMENTAL & SCIENCE EDUCATION
2016, VOL. 11, NO. 12, 5221-5236

Section Height Determination Methods of the Isotopographic Surface in a Complex Terrain Relief

Guldana D. Syzdykova[a] and Azimhan K. Kurmankozhaev[a]

[a]Kazakh National Technical University named after K.I. Satpaev, Almaty, KAZAKHSTAN

ABSTRACT

A new method for determining the vertical interval of isotopographic surfaces on rugged terrain was developed. The method is based on the concept of determining the differentiated size of the vertical interval using spatial-statistical properties inherent in the modal characteristic, the degree of variability of apical heights and the chosen map scale. It was found that the morphometric characteristics of the terrain are highly informative, can serve as geoindicators, and have applied value; their calculation formulas were provided. An analytical assessment of the determination of differentiated sizes of the vertical interval was made. Its initial parameters are as follows: modal height, scale and variability of apical heights. The vertical intervals are differentiated by dividing the morphometric field of the terrain into two parts, the height in the contours whereof is lower (hi < hmo) and higher (hi > hmo) than the modal height, respectively; the reasoning behind the analytical assessment of the calculation of apical height variability and vertical intervals, which takes into account the peculiarities of terrain formation, was given; the main contour is drawn through the modal value of apical heights and then drawn through other contour systems based on the calculated vertical interval values, differentiated by the divided parts of the morphometric field.

KEYWORDS
Geodesy, vertical interval of the terrain, isotopographic surface, morphometric fields, isogeometric analysis

ARTICLE HISTORY
Received 10 March 2016
Revised 20 April 2016
Accepted 28 April 2016

Introduction

Topographic and cartographic studies have begun effectively using isogeometric analysis methods in the form of cartographic and spatial contour lines, which have theoretical and practical advantages compared to one-

CORRESPONDENCE Guldana D. Syzdykova ✉ guldanasd@mail.ru

dimensional analytical assessments (Blogov & Belopasov, 1974; Tomlin, 2013; Manual of Surveying Instructions, 2009).

It is pertinent to point out that a contour line model (map, chart, etc.) reflects the surface more effectively and to a greater extent, compared to a series of inter-object lines, and gives ground for more hypotheses than a simple numerical assessment of the match between the mathematical surface and its real prototype (Triebel, Pfaff & Burgard, 2006). The depiction of the surface terrain using contour lines is a mathematically reasonable technique that graphically reflects the main forms of terrain and highlights their typical morphometric peculiarities (Chung & Fabbri, 1999). The latter is achieved primarily by choosing the optimal vertical interval, and drawing additional contours and conditional figures (Song et al., 2008).

The effectiveness of different quantitative and qualitative maps, charts, and other geometric models affects the reliability and quality of the results of structural and geometric modeling of the spatial arrangement of mineral resource signs and directly affect the assessment of mineral resource deposits (Vallée & Sinclair, 1998). The international practice of assessing the resource potential of minerals is based on the quality and reliability of mapping (CIM Standards on Mineral Resources and Reserves – Definitions and Guidelines, 2000; Standards of Disclosure for Mineral Projects, 2000; Exploration Best Practice Guidelines, 2000). With that, the main modeling parameter is the vertical interval, i.e. the variation between the integral-valued marks of two neighboring points of a graded projected line.

The vertical interval affects the quality, level of detail, clearness, cost, and reliability of topographic maps and isogeometric charts. Moreover, it is an important parameter in static geomodeling (Singer & Menzie, 2010).

Literature Review

In accordance with the current Manuals and Guidelines for topographic surveys in CIS states, the drawing of standard vertical intervals of the terrain mostly uses the classic formula that is derived from the basic morphometric parameters of the terrain (Manual for Topographic Surveys at a Scale of 1:5000, 1:2000, 1:1000, 1:500, 1985; Fundamental Principles for Drawing and Updating Topographic Maps at a scale of 1:10000, 1:25000, 1:50000, 1:100000, 1:200000, 1:500000, 1:1000000, 2005):

$$h = atg\beta \tag{1}$$

where a is the distance between contour lines on the map (horizontal equivalent); β is the slope of the terrain.

Equation (1) is used to determine the values of vertical intervals on topographic maps, depending on the nature of the terrain, and to divide it into plain terrain with a slope of β=0-2°, hilly terrain with a slope of β=2-4°, rugged terrain with a slope of β=4-6°; mountainous or piedmont with a slope of β>6°.

The classic equation (1) is also used in its modified form when drawing topographic maps and charts (Vilesov, 1973):

$$\begin{cases} h = \frac{S}{K1000} ctg\beta \\ h = \frac{S}{1000} \alpha tg\beta \end{cases} \tag{2}$$

where S is the scale of the chart; K is the number of contour lines on the map, drawn on a section of a straight 1 mm line; a is the horizontal equivalent.

Table 1 shows the normal contour intervals (a=0.2 mm; k=1) for topographic maps with a scale of 1:5000 – 1:100000, as well as the data on the vertical interval provided by the Chief Directorate of Geodesy and Cartography (Manual for Topographic Surveys at a scale of 1:5000, 1:2000, 1:1000, 1:500, 1985).

Table 1. Established sizes of the vertical interval

Map scale	Vertical interval, m 45%		
1:5000	1	0.5-0.5	0.25-10.0
1:10000	2	2.5-2.5	0.25-10.0
1:25000	5	2.5-5.0-10.0	1.0-10.0
1:50000	10	100.0-20.0	10.0-20.0
1:100000	20	20.0-40.0	10.0-40.0

The second to last column shows the vertical interval on topographic maps calculated at υ=45°, while the last column – mostly on special maps. The data show that the vertical interval values may be used to determine the vertical interval on topographic maps, but cannot be used on special maps, since they are used in engineering estimations. It is customary first to choose the vertical interval that guarantees the accuracy of engineering estimations and then to set the survey scale based on that.

Studies show that in the geometrization of quantitative indexes of deposits, this classic equation is also used as a basic assessment of the contour interval in the following form (Triebel, Pfaff & Burgard, 2006; Vilesov, 1973):

$$h = i \cdot \ell \left(i = \frac{h}{\ell} \right), \tag{3}$$

where i is the slope; ℓ is the horizontal equivalent.

It is worth noting that these classic equations take into account the functional relation between the vertical interval and the slope and distance between characteristic points (horizontal equivalent) of the terrain; thus, it serves as a basis for assessing the vertical interval (Song et al., 2008). These equations were obtained based on an analogue of a right triangle. However, such straight lines are nonexistent on the real Earth’s surface, which means that these equations cannot convey the actual morphometrics of the terrain. This fact becomes obvious, considering the dynamic of changes in the slope and elementary forms of terrain surface, which mostly determine the vertical interval (Viduyev & Polischuk, 1973).

Scholars argue that unlike CIS states, which mostly use one vertical interval, most countries set at least two vertical intervals for topographic maps of the same scale (Blogov & Belopasov, 1974; Chung & Fabbri, 1999). However, this does not imply that the optimal value of the vertical interval is achieved simply by differentiating it. The determination of the vertical interval of the terrain remains problematic due to the variety, different importance, and number of factors that affects its accuracy. The promising differentiated approach to determining the vertical interval for the topographic base requires thorough substantiation; the conventional technique for choosing the interval has several flaws that often cause accumulation and increase of labor costs. This practice generally causes mismatches between the depicted and actual surface, misperception of the level of detail and accuracy of the isotopographic maps, and false conclusions and accumulation of suboptimal solutions.

Aim of the Study

The purpose of the study is to elaborate a method of determining the vertical interval of isotopographic surfaces on rugged terrain.

Research questions

What are the essential requirements for differentiated sizes of the vertical interval?

Method

The suggested method is based on the concept of determination of the differentiated sizes of the vertical interval using the properties of the main informative and geoindicator characteristics of apical heights, which take into consideration the morphometric peculiarities of the terrain. The essence of the method lies in substantiating the effective values of the vertical interval according to structurally differentiated sections of the morphometric field of the terrain, which reflect various sets of actual vertical interval values in this area.

It was taken into consideration that the terrain as a random field of heights is a hidden topographic surface, which is revealed only in certain nodes with random values across the area. Therein, an individual structural parameter may be more hidden, which is caused by the “consistent” formation of the attribute’s distribution structure; thus, it may serve as a natural and adequate characteristic of the terrain attribute distribution.

The differentiation of the vertical interval is based on the concept of geometrical division of the morphometric field of heights into separate structural parts with different absolute values of apical heights, which are distinguished with respect to the single modal value of the surface height. Thus, three main optimized sizes of the vertical interval are distinguished, which are geometrized through the modal, below-modal, and above-modal values of terrain heights. The modal terrain height is used as a structural regulator that divides the morphometric field into several structural sections with different absolute values and degrees of variability of apical heights.

The area of differentiation of the vertical interval in the space of the morphometric field of the terrain is written as follows:

$$\begin{cases} h_1 \in Q_{(h_{m0})}, S_1 = h_{mo} \\ h_2 \in Q_{(h_i<h_{m0})}, S_2 = \sum_{h_{min}}^{h_{mo}} h_2^i \\ h_3 \in Q_{(h_i>h_{m0})}, S_3 = \sum_{h_{mo}}^{h_{max}} h_{3,}^i \end{cases} \tag{4}$$

where h_{mo}, h_{min}, h_{max} are the modal, minimum, and maximum values of apical heights; h_1, h_2, h_3 are estimated effective values of the vertical interval set on a case-by-case basis for three distinguished parts of the morphometric field of the terrain; $Q_{(h_{m0})}$, $Q_{(h_i<h_{m0})}$, $Q_{(h_i>h_{m0})}$ are geometric areas of distribution of apical heights with $h_i \approx h_{m0}$, $h_i < h_{m0}$, $h_i > h_{m0}$; S_1, S_2, S_3 is the sum of absolute values of heights, respectively, for three distinguished parts of the morphometric field of the terrain.

The nature of changes of h_{mo}, h_l, h_h in the following conditions was determined:

$h_l - h_{mo} \approx h_h - h_{mo}$ was found in plains, including flat, lofty, billowy, and hilly terrain;

$h_l - h_{mo} < 0, h_h - h_{mo} > 0$ was found in hilly terrain, including large, medium, and small hills;

$h_l - h_{mo} < 0, h_h - h_{mo} > 0$ was found in mountainous terrain, including high, medium, and low mountains.

In cases when the empirical distribution of terrain height is symmetrical – $h_{mo} \approx h_{av}$; in cases when the distribution of asymmetrical – $h_{mo} < h_{av}$.

Data, Analysis, and Results

The analytical framework of the assessment of the vertical interval in accordance with the offered method is presented as a system of assessments in the following form: $\varphi(h_0) = f(h_{mo})$; $\varphi(h_l) = f(h_{mo}\gamma_l, \mathrm{M})$; $\varphi(h_h) = f(h_{mo}\gamma_h, M)$

$$\begin{cases} h_o = h_{mo} \\ h_l = h_{mo} - \gamma_l\left(\frac{\mathrm{M}}{1000}\right), \\ h_h = h_{mo} + \gamma_h\left(\frac{\mathrm{M}}{1000}\right) \end{cases} \quad (5)$$

where h_{mo} is the modal value of apical heights; γ_l and γ_h are the indexes of height variability, the values whereof are lower and higher, respectively, than the modal height of the terrain, unit fractions; M is the denominator of the numerical scale; h_l is the sought vertical interval for areas, within the contours of which the terrain height does not exceed the modal height ($h_l < h_{mo}$); h_h^i is the sought vertical interval for areas, within the contours of which the terrain height exceeds the modal height ($h_h > h_{mo}$). In this case, apical heights mean certain terrain points that are higher than the plain of the lower denudation level.

The modal value is found with a histogram, through calculations or visually from the observed characteristics of the distribution of terrain heights across the area. With a sufficient amount of information, the empirical value of the mode is found from a histogram or an ordered sample with the following equation:

$$X_{mo} = x_{o_{l.b.}} + \Delta x \frac{R_{\mathrm{MO}} - R_{\mathrm{MO}-1}}{(R_{\mathrm{MO}} - R_{\mathrm{MO}-1}) + (R_{\mathrm{MO}} - R_{\mathrm{MO}+1})}, \quad (6)$$

where $x_{o_{l.b.}}$ is the lower boundary of the modal interval; R_{MO} is the rate of the modal interval; R_{MO-1} is the rate of the interval preceding the modal one; R_{MO+1} is the rate of the interval that follows the modal one; $\Delta x = h$ is the interval variability.

The formulas of dependency between the mode and the arithmetic mean ($\bar{x}$), the median (Me), and the dispersion (σ^2) are as follows (Viduyev & Polischuk, 1973):

$$\begin{cases} A = \frac{\bar{x} - X_{mo}}{\sigma} \\ X_{mo} = \frac{S(S+1)}{[(S-1)^2 - \sigma S|\bar{x}|]} \end{cases} \quad (7)$$

where $\bar{x}$ is the arithmetic mean; A is asymmetry; S is the sum of observations; σ^2 is dispersion.

The Pearson correlation is a follows:

$X_{mo} = \bar{X} - 3(\bar{X} - Me).$ (8)

The Kelly formula is as follows:

$X_{mo} = \bar{X} - \frac{\bar{x} - Me}{c}$, C is the constant (9)

The calculation formulas for the mode were drawn in accordance with the theoretical distribution of probabilities (Kurmankozhaev, 2013):

with normal distribution, the mode equals the mean

$X_{mo} = \bar{X}$; (10)

with lognormal distribution

$X_{mo} = \frac{1}{n} \sum lgX 10^{-\frac{\sigma^2}{m}}$; (11)

with gamma distribution

$X_{mo} = \alpha \cdot \beta \frac{\sigma_x^2}{(\alpha+1)}$. (12)

Here $\alpha = \left(\frac{M_x}{\sigma_x}\right)^2 - 1$; $\beta = \frac{\sigma_x^2}{M_x}$; $M_x = \beta(\alpha + 1)$, where α, β are theoretical parameters of gamma distribution; M_x is the mean value:

with Pearson type V distribution

$X_{mo} = \frac{V}{P'}$, (13)

where V, P are the Pearson distribution parameters;

with probability-structural distribution

$X_{mo} = \bar{x}_m - \frac{d_2 th(x_2 - x_0) - d_1 th(x_1 - x_0)}{th(x_2 - x_0) - th(x_1 - x_0)}$ (14)

where X_{mo}, x_2, x_1 are the modal, maximum, and minimum values of the attribute; thx is the hyperbolic tangent; $d_2 = x_2 - X_{mo}$, $d_1 = x_1 - X_{mo}$.

The relation of the mode to the variability indexes (r=0.60-0.70) is also seen from the statistical ensembles by the geomechanical attribute of durability (N=115) of chromite minerals in the following form:

$$\begin{cases} X_{mo} = 1{,}482 \exp(0.638\sigma), \eta = 0.61 \\ X_{mo} = 0.62 \exp(0.601d), \eta = 0.49 \\ X_{mo} = 0.81\sigma + 0.04d - 0.78, R = 0.76 \end{cases}, \quad (15)$$

where σ is the standard deviation; d is the range; η, R is correlation coefficients.

The main parameter of the model assessment that takes into account the variability of terrain heights when differentiating the contour interval is the structural indicator that reflects the peculiarities of the geometry of elementary terrain surfaces through successive differences.

The new analytical assessment of the effect of variability and degree of geometric ruggedness (curves) of the terrain surface on the vertical interval was developed in the form of a dimensionless parameter (γ) that is expressed via sums of successive differences of terrain heights. The geometric variability of apical heights equals the sum of first-order successive absolute differences of neighboring apical heights arriving at the unit of their length in the studied morphometric field of the terrain:

$\gamma_k = \frac{1}{L} \sum_{i=1}^{n} |\Delta'|$. (21)

For terrain surfaces with apical heights that are lower or higher than the modal height, formula (20) acquires the following form:

$$\begin{cases} \gamma_l = \frac{1}{L_l}\sum_1^k|\Delta'| \\ \gamma_h = \frac{1}{L_h}\sum_{i=1}^{n-k}|\Delta'| \end{cases} \quad (22)$$

where γ_l, γ_h are coefficients that reflect the geometrical variability in the parts of the terrain surface where $h_i < h_{mo}$ and $h_i > h_{mo}$, respectively; L_l, L_h are mean parametrized lengths of the terrain surface in the parts where $h_i < h_{mo}$ and $h_i > h_{mo}$, respectively; $\sum_{i=1}^k|\Delta'|$, $\sum_{i=1}^{n-k}|\Delta'|$ is the sum of absolute first-order successive differences of terrain heights in the parts where $h_i < h_{mo}$ and $h_i > h_{mo}$, respectively; k, (n-k) are the numbers of first-order differences in the parts the terrain where $h_i < h_{mo}$ and $h_i > h_{mo}$; n is the total number of first-order differences across the entire terrain surface.

The random location of apical heights is expressed through the mean value of first-order successive differences as follows:

$$\bar{\Delta}' = \frac{1}{n-1}\sum_{i=1}^n|\Delta_i'| = \frac{1}{n-1}\sum_i^n|\Delta h_i'|, \quad (23)$$

where Δ_i' is the absolute i value of first-order successive differences; $\Delta h_i'$ is the absolute value of first-order differences for terrain height excesses.

The mean value of first-order successive differences of apical heights, the sizes whereof exceed the modal height ($h_i > h_{mo}$):

$$\bar{\Delta}_h^1 = \frac{1}{k-1}\sum_{i=1}^k(H_i^h - H_{i+1}^h)' \ . \quad (24)$$

The mean value of first-order successive differences of apical heights, the sizes whereof do not exceed the modal height:

$$\bar{\Delta}_l^1 = \frac{1}{n-k}\sum_{i=1}^{n-k}(H_j^l - H_{j+1}^l)' \ , \quad (25)$$

where H_i^h, H_j^l are the sizes of apical heights when $H_i^h > H_{mo}$ and $H_j^l < H_{mo}$, respectively.

The dependency between the recommended index of height variability (γ_i) and terrain slope (β) results from their geometrical connection; it is expressed as follows:

$$\begin{cases} \gamma_L = \frac{1}{L_L}\sum_1^k l_{L_i} tg\beta_{L_i} \\ \gamma_H = \frac{1}{L_H}\sum_1^{n-k} l_{H_i} tg\beta_{H_i} \end{cases}, \quad (26)$$

where l_{L_i}, l_{H_i} is the distance between neighboring height values for the lower ($h_i < h_{mo}$) and higher ($h_i > h_{mo}$) parts of the terrain surface, respectively.

The conclusion is that using the concept of modal characteristics (x_{mo}) and amplitude of location variability (γ_i) of terrain heights as the main spatial-statistical parameters when creating a composite structure of the modal assessment of the vertical interval is innovative and reasonable. The γ parameters tells the presence of geometrical variability; if the amplitude fluctuation of terrain heights is entirely random, then the value of this coefficient $\gamma \approx max$, i.e. shows the presence of variability, if vice versa, then $\gamma \approx min$.

The results of the calculation of differentiated sizes of the vertical interval in accordance with the developed method were obtained for comparative analysis from three natural-experimental objects selected in different regions of Kazakhstan. The first object is an area in the Jambyl Region, the data for which were obtained from the results of a survey and a 1:500 chart. The terrain of the object is plain; the variation coefficient of the elementary terrain surface height does not exceed 40-42%; the mean height of elementary surfaces h_{me} = 6.2 m; the amplitude range of heights d = 28.2 m. The second object is an area in the Glubokoye District, the data for which were obtained from the results of a survey and a 1:1000 chart. The terrain of the object is hilly; the variation coefficient of the elementary terrain surface height v = 63%; the mean height of elementary surfaces h_{me} = 10.3 m; the amplitude range of heights d = 3.76 m. The third object is an area in the Jualy District. The terrain of the object is piedmont; the variation coefficient of the elementary terrain surface height is 72.5%; the mean height of elementary surfaces h_{me} = 12.3 m; the amplitude range of heights d = 57.2 m; the scale of the survey and topographic chart is 1:2000.

The technological order of the method execution includes three stages of contour drawing. During the initial basic stage, the main contour is drawn on the isosurface of the terrain. The main contour is drawn according to the modal value of apical heights that covers at least 40-50% of all values of apical heights. This is confirmed by the abovementioned facts, since statistical distributions of terrain heights are described by an extremely asymmetrical radial distribution, when about 50% of all sets of terrain height values are concentrated in the modal value.

The high informative value, potential reliability of detection, and other abovementioned properties of the apical height mode allow accepting the contour that runs through its value as the main contour of the terrain isosurface.

Thus, the conventional theory that the main contours should run through the typical points of the terrain acquires a more reasonable and substantial meaning.

The values of apical heights that are close to the modal height should be averaged to draw the main contour. To that end, the recommendation is to use a mean arithmetic technique of the moving average according to the following formula (Vilesov, 1973):

$$\Phi = (xy) = \frac{1}{n}\sum_{i=1}^{n} h_{i,} \tag{27}$$

where n is the number of averaged groups of apical heights, the values whereof are close to the modal height.

The modal value of terrain heights at the three natural-experimental objects was found from the data of ordered samples of empirical distributions of apical heights (Table 2). The ordered samples of height distribution in these objects were taken with conventional techniques from the statistical ensembles of actual values of apical heights (excesses), calculated based on topographic surveys and charts of various scales (1:500, 1:1000, 1:2000).

The following stages of the method execution include the structural differentiation, during which the developed analytical assessment is used to find the sought vertical intervals in the distinguished parts of the morphometric field, where the values of apical heights are lower ($h_i < h_{m0}$) and higher ($h_i < h_{m0}$) than their modal value, respectively. Contours are drawn along both parts of the morphometric field according to the set vertical intervals with conventional techniques.

Table 2. Collective results of the variation sets of empirical distributions of apical heights for the three natural-experimental objects

№	Plain-hilly terrain, topographic survey at a scale of 1:500			Hilly terrain, topographic survey at a scale of 1:1000			Piedmont terrain, topographic survey at a scale of 1:2000		
	Classes, m.	Mean, m.	Frequency, un. fr.	Classes, m.	Mean, m.	Frequency, un. fr.	Classes, m.	Mean, m.	Frequency, un. fr.
1	0.1-0.6	0.3	34	0.1-0.5	0.25	17	0.1-0.5	0.25	16
2	0.6-1.2	0.9	31	0.5-1.0	0.75	13	0.5-1.0	0.75	15
3	1.2-1.8	1.5	22	1.0-1.5	1.25	10	1.0-1.5	1.25	5
4	1.8-2.4	2.1	21	1.5-2.0	1.75	11	1.5-2.0	1.75	6
5	2.4-3.0	2.7	10	2.0-2.5	2.25	5	2.0-2.5	2.25	2
6	3.0-3.6	3.3	6	2.5-3.0	2.75	2	2.5-3.0	2.75	5
7	3.6-4.2	3.9	3	3.0-3.5	3.25	3	3.0-3.5	3.25	2
8	4.2-4.8	4.5	2	3.5-40	3.75	2	3.5-4.0	3.75	1
	N=129, h_{mo}=0,64			N=63, h_{mo}=0,50			N=52, h_{mo}=0,67		

For the first object, the topographic chart is large-scale, while the terrain category is plain; for the second object – the topographic chart is small-scale, while the terrain category is hilly; for the third object, the topographic chart is medium-scale, while the terrain category is medium-hilly piedmont. Therefore, the degree of variability and fluctuation of the apical heights of elementary terrain surfaces in these objects are different. The results of calculation of the differentiated sizes of the vertical interval, conducted according to the modal assessment for the three natural-experimental objects, are presented in Table 3.

Table 3. Results of calculation of the vertical interval according to the recommended methods for areas with different scales and terrain

№	Objects and scales	Modal value, h_{m0}, m	Number of vertical intervals for elementary terrain surfaces	Lengths of design profiles for the terrain surfaces, m	Sum of first-order differences, m.	Variability indexes of terrain heights, un. fr.	Set vertical intervals, m.

			N_L	N_H	L_L	L_H	$\sum_1^n \Delta'_I$	$\sum_1^{ne} \Delta'_I$	γ_v	γ_H	h_o	h_l	h_h
1	Plains (Jambyl Region, 1:500)	0.64	52	97	3328	6208	24.15	425.4	0.007	0.064	0.75	0.36 $к_о=2$	0.40 $к_о=15$
2	Hills (Glubokoye District, 1:1000)	0.52	25	63	2400	6048	15.0	229.2	0.010	0.037	0.55	0.49 $к_о=2$	0.52 $к_о=19$
3	Piedmonts (Jualy District, 1:2000)	0.67	17	75	1360	6000	8.5	360.0	0.006	0.09	0.67	0.95 $к_о=2$	1.3 $к_о=8$

A total of 20 contours will be drawn on the isotopographic chart on a scale of 1:500, including 1 – main contour with a vertical interval h_o = 0.64 m, 2 – contours with a vertical interval lower than the modal height with h_L = 0.39 m, 15 – contours with a vertical interval higher than the modal height with h_H = 0.25 m. A total of 21 contours will be drawn on the isotopographic chart on a scale of 1:1000, including 2 contours along the lower part of the terrain with h_L = 0.49 m, 19 contours along the higher part of the terrain with h_H = 0.52 m, and 1 main contour with a vertical interval h_0 = 0.55. A total of 20 contours will be drawn on the isotopographic chart on a scale of 1:2000, including 1 main contour (h_o = 0.67 m), 2 contours along the lower part of the terrain with a vertical interval h_L = 0.95 m, and 8 contours along the higher part of the terrain with a vertical interval h_H = 1.3 m.

The differentiated vertical intervals of the three natural-experimental types of terrain and different scales, obtained according to the developed method, were compared. For similar objects with identical scales and slopes, found in accordance with the current manual, the assessment was conducted with regard to the known values of vertical intervals (Table 4).

The study used estimated results of the vertical interval assessment based on the recommended method (Table 3) and the vertical intervals found in the tabular scale set in the current manual (Fundamental Principles for Drawing and Updating Topographic Maps at a scale of 1:10000, 1:25000, 1:50000, 1:100000, 1:200000, 1:500000, 1:1000000, 2005).

Table 4. Collective results of the estimation of vertical intervals according to the tabular scale and calculated based on the recommended method

Terrain with dominating slope angles	Survey scale			Survey scale		
	1:500	1:2000	1:1000 (1:500)	1:500	1:2000	1:1000
	Vertical intervals according to instructions, m			Vertical intervals found based on the recommended method, m (h_{mo}, h_L, h_H)		

Plains with slope angles up to 2°	0.5	0.5	0.5	0.64; 0.40; 0.36
		(1.0)		
Hills with slope angles up to 4°	0.5	0.5	0.5	
		1.0		
Rugged terrains with slope angles up to 6°	0.5	2.0	0.5	0.55; 0.49; 0.52
	1.0	(1.0)	1.0	
Mountainous and piedmont terrain with slope angles over 6°	1.0	2.0	1.0	0.67; 0.95; 1.3
		2.0		

The comparative assessment showed the following:

1) The differentiated values of vertical intervals, obtained with the recommended method for plain (0.54 and 0.36; 0.40), rugged hilly (0.55; 0.49; 0.52), and piedmont (0.67; 0.95; 1.3) terrain do not exceed the intervals set by the Federal Agency of Geodesy and Cartography of the Russian Federation (Fundamental Principles for Drawing and Updating Topographic Maps at a scale of 1:10000, 1:25000, 1:50000, 1:100000, 1:200000, 1:500000, 1:1000000, 2005) for large-scale topographic maps (0.5 ÷ 5.0 m) or the ones often used for vertical interval maps (0.25 ÷ 10.0 m) set in accordance with manuals; however, they differ significantly in different differentiated ranges.

2) Changes in the differentiated sizes of the vertical interval are inversely proportional to the variability amplitude of apical heights (γ_L, γ_H), the high values whereof correspond to small vertical intervals and vice versa.

3) The ratio of the distribution of modal height values in sets of values of the natural-experimental objects ranged from 48 to 61%; smaller values of modal height correspond to smaller values of apical height variability and smaller sizes of the vertical interval, and vice versa; this directly proportional relation is found in all three natural-experimental objects; these regularities do not contradict the abovementioned analytical assessments of their interrelation.

4) The effect of the topographic base scale (1:500, 1:1000, 1:2000) on the sizes of the vertical interval is proportional; it has varying significance, depending on the variability of the terrain heights (γ_i).

Discussion and Conclusion

The elaboration of the theory of assessment of dependent observations is related to successive differences and is widely used in practice. The squares of first-order successive differences were used by Ye. I. Azbel (1976) to assess the dispersion of a set of observations that has a regular constituent

$$\sigma_1 = \sqrt{\frac{1}{2(n-1)}\sum_{i=1}^{n-1}(x_i - x_{i+1})^2}\ . \tag{16}$$

This formula was used by Yu.V. Linnik and A.P. Khusu (1958) to assess the ruggedness of ground profile. In order to detect corrugations, they used the ratio of the sum of squares of successive differences $(\Delta')^2$ to dispersion in the following form:

$$r = \frac{1}{2(n-1)}\sum_{i=1}^{n-1}(h_{i+1} - h_i)^2 / \frac{1}{n-1}\sum_{i=1}^{n}(h_i - \bar{h}_{cp})^2 \ . \quad (17)$$

In order to assess the accuracy of the hypsometric chart, Popov B.I. used the squares of the second-order successive differences in the following form (Kamorny & Koscheva, 1981):

$$\sigma_2 = \sqrt{\frac{1}{6(n-1)}\sum_{i=1}^{n-2}(x_i - 2x_{i+1} + x_{i+1})^2} \ . \quad (18)$$

Successive first- and second-order differences were used to assess the geometry of the terrain surface as a random number in the form of a sum of their squares (Neumyvakin, 1976). These differences are mostly used in the following form:

$$\sigma_{cm} = \sqrt{\frac{1}{2(n-1)}\sum_{i}^{n}\left|\Delta_i'\right|^2} \ , \quad (19)$$

where Δ' are the first-order differences of heights at i and (i+1) points; n is the number of points (peaks).

V.M. Gudkov (1979) offers formulas expressed through sums of squares of first-order differences to characterize the general smoothing of the (S_o) equation of ore and rock contact in deposits at specific distances:

$$S_o = \frac{1}{4n}\sum_{i=1}^{n}(h_i + h_{i+1})^2. \quad (20)$$

The above analytical assessments (equations 15-19) show that the first- and second-order successive differences are suitable for assessing the variability of attributes of a geometrical object; they reveal the nature of amplitude fluctuations for set observation points. The absolute sum of first-order successive differences in the terrain field assesses the sum of detected fluctuation amplitudes and increases linearly with an increase in the amplitude. The choice of the variability characteristic based on first-order successive differences when differentiating vertical intervals in order to ensure the accuracy of the assessment is reasonable.

Also can add that the vertical interval on modern topographic maps varies significantly due to different types of terrain, the lack of standard requirements to topographic maps, and the peculiarities of the development of cartography in this or that country (Chentsov, 1956; Kneissl, 1957; Vermessungswesen, 1953).

In Italy, USA, and Canada, topographic maps of the same scale have at least two vertical intervals, while in countries with different types of terrain, such maps have 3-4 or more (Manual of Surveying Instructions: For the Survey of the Public Lands of the United States, 2009; Manual of Instructions for the Survey of Canada Lands, 1996). Even in small countries (Belgium, the Netherlands), large-scale maps (1:20000, 1:25000) have two vertical intervals, depending on the nature of the terrain in this or that area (de Leeuw, 2008). In England, auxiliary or approximate contours are widely used for maps of the same scale with a single accepted vertical interval; in Belgium, Denmark, and some other countries, on 1:25000 maps and ones with a similar scale, regions with a plain terrain have a vertical interval of 0.3-2.5.

Similar solutions are used in large-scale mapping of deposits and quarries in India, Australia, Central and Southern African states (David, 1997; Dominy et al., 1997). In different countries, 1:200000 (1:250000, 1:252440) maps have different purposes, which is why the range of used vertical intervals is considerable – from 7.6 to 305 m; 20 m and less – on topographic maps of plains and moderately rugged regions (the Netherlands, France); 25-50 m – on maps of countries with rugged and mountainous terrain; 60 m and more on reconnaissance maps (underexplored regions of the USA, Canada) (Manual of Surveying Instructions: For the Survey of the Public Lands of the United States, 2009; Manual of Instructions for the Survey of Canada Lands, 1996). The techniques for determining the vertical interval used in Germany are somewhat different – they use the classic equation of the geometric relation between triangle sides to determine the normal interval (Chentsov, 1956).

To sum up, the newly developed method for determining the vertical interval enables differentiating its sizes by discretely distinguished land plots, which in turn provides for accurate and optimal parameters of topographic and cartographic maps and charts.

The method contains analytical assessments of the determination of differentiated sizes of the vertical interval, the structural initial parameters of which are the main natural spatial-statistical characteristics of the morphometric field of the Earth's surface.

The concept of using the modal characteristic and amplitude variability of terrain height location as the main spatial-statistical parameters is innovative and applicable to the determination of the vertical interval. The main statistical-geometric characteristic of the morphometric field is the modal height of the terrain; it has high informative value (48-60%), unbiasedness during assessment, real quantitative reliability, and special geometrical-statistical properties that form the typical nonhomogeneous parts of the morphometric field of the terrain. The spatial characteristics of the terrain morphometrics are geometric elements (prolongation length, absolute sizes, amplitude variability, and difference range) of the apical heights, which are structural components of the developed analytical assessment of the height variability determination. These structural components provide for an accurate and rational differentiation of the vertical interval. This analytical assessment of the recommended method that is part of the model structure reflects the degree of amplitude variability of typical natural heights, depressions (ravines, etc.) and plains, with regard to the selected scale and spatial length of location and changes in the values of terrain heights.

The accuracy of isotopographic maps and charts, mathematical and isogeometric charts, and reliability of their results when used according to the developed method may be achieved by using differentiated sizes of vertical intervals by drawing a contour system in the form of a single main contour along the modal height across the parts of the morphometric field, the apical heights whereof are lower and higher than the modal height; the most acceptable accuracy characteristics of the reliability of the sought vertical interval that are used in many studies are the mean squared error, random errors, interpolation error when drawing contours, and the terrain generalization error.

Thus, the differentiated sizes of the vertical interval should meet the following requirements:

- the vertical interval should be greater than the minimum horizontal interval;

- the accuracy of estimation of the volume of earthwork and other development and research works should not exceed the accuracy when using accepted vertical intervals;

- the vertical interval should not exceed the error of determination of the point marks on the depicted topographic function;

- the distance between the contours should ensure proper visualization and legibility of the chart;

- the vertical interval of the topographic functions should comply with the accuracy of the initial data and set boundaries;

- the selection of the toposurface section should be based on the correspondence between the degree of certainty of the function and the accuracy of the image;

- it is not necessary to use only a single calculation formula to assess the vertical interval.

The comparative assessment of the recommended method was conducted by calculating the differentiated sizes of the vertical interval and accuracy of its determination in three natural-experimental areas of different scales and terrain type. The results confirmed:

- the accuracy of estimated sizes of the vertical interval, differentiated in accordance with the recommended method for isotopographic charts with scales 1:500, 1:1000, and 1:2000, and their comparability to the sizes of the vertical interval found in the manuals and experience of cartographic works;

- the ability to increase the level of accuracy, detail, visualization, and convenience of isotopographic maps and charts when using differentiated sizes of the vertical interval determined in accordance with the recommended method.

It is worth noting that the creation of a rational analytical framework for assessing the main morphometric parameters of the terrain not only increases the effectiveness of topographic and cartographic products, but also is required in a number of engineering fields that use information about terrain: in the construction of roads, canals, telecommunication lines, in the design of aircraft control systems, and other fields of engineering. Increasing the reliability of toposurface mapping improves the quality of geomodeling, which in turn improves the quality of assessment of mineral deposit resources.

Implications and Recommendations

The practical value is that the basic formulas for calculating the vertical interval of the terrain and assessing the line that reflects the dependency of the vertical interval on the slope and horizontal equivalent were suggested. The further work on the research involves the examination of the proposed method on several projects in order to reveal its advantages and disadvantages.

Disclosure statement

No potential conflict of interest was reported by the authors.

Notes on contributors

Guldana D. Syzdykova holds a PhD of Department of Mine Survey and Geodesy, Kazakh National Technical University named after K.I. Satpaev, Almaty, Kazakhstan.

Azimhan K. Kurmankozhaev holds a Doctor of Science, Professor of Department of Mine Survey and Geodesy, Kazakh National Technical University named after K.I. Satpaev, Almaty, Kazakhstan.

References

Azbel, Ye. I. (1976). The Peculiarities of Studying the Statistical Structure of Variability of Ore Quality in Deposits. *Mine Survey and Geodesy, 3,* 95-107.

Blogov, I. F. & Belopasov, A. A. (1974). On Topographic Survey Scales for Design and Construction. *Problems of Engineering Geodesy in Construction, 3*, 17-20.

Chentsov, V. N. (1956). Vertical intervals on foreign topographic maps. *Geodesy and Cartography, 3*, 59-63.

Chung, C. J. & Fabbri, A. G. (1999). Probabilistic Prediction Models for Landslide Hazard Mapping. *Photogrammetric Engineering and Remote Sensing, 65(12),* 1389-1399.

CIM Standards on Mineral Resources and Reserves – Definitions and Guidelines. (2000). Prepared by the CIM Standing Committee on Reserve Definitions, October. *CIM Bulletin, 93(1044),* 53-61

David, M. (1997). *Geostatistical Ore Reserve Estimation*. Amsterdam: Elsevier Scientific Publishing, 364 p.

de Leeuw, E. D. (2008). *International Handbook of Survey Methodology.* New York: Taylor & Francis, 558 p.

Dominy, S. C., Annels, A. E., Camm, G. S., Wheeler, P. & Barr, S. P. (1997). Geology in the Resource and Reserve Estimation of Narrow Vein Deposits. *Exploration and Mining Geology, 6*, 317–333.

Exploration Best Practice Guidelines. (2000). Prepared by the CIM Standing Committee on Reserve Definitions. *CIM Bulletin, 93(1044),* 53-61.

Fundamental Principles for Drawing and Updating Topographic Maps at a scale of 1:10000, 1:25000, 1:50000, 1:100000, 1:200000, 1:500000, 1:1000000. (2005). GCIRR-02-006-05 *Agency for Land Resource Management of the Republic of Kazakhstan*. Astana, Kazakshtan: *Geodesic, Cartographic Instructions, Regulations and Rules*, 46 p.

Gudkov, V. M. (1979). *Study of Variability and Distribution of the Main Attributes of Minerals*: PhD Abstract. Moscow, 32 p.

Kamorny, V. M. & Koscheva, A. I. (1981). Determining the Parameters and Assessing the Accuracy and Location of Contours when Conducting a Topographic Survey of Continental Shelves. *Works of the Central Research Institute of Geodesy, Aerial Surveying and Cartography*, *277*, 22-28.

Kneissl, J. E. (1957). Handbuch der Vermessungskunde. *Wichmann Verlag, Heidelberg*, *3*, 53-60.

Kurmankozhaev, A. (2013). Theory of Regulation of Stabilization of Quality of ore Products During Production. *Asia-Pacific Congress for Computational Mechanics* (APCOM 2013). Singapore, December.

Liisclmer, H. (1953). Zeitschrift rur Vermessungswesen. *Herausgeber und Verleger*, *12(1903),* 143-154.

Linnik, Yu. V. & Khusu, A. P. (1958). Some Considerations Regarding Statistical Analysis of the Surfaces of Ground Profile. Interchangeability and Measurement Equipment in Machine Building. *M.: Machine Building*, 65-69.

Manual for Topographic Surveys at a scale of 1:5000, 1:2000, 1:1000, 1:500. (1985). Moscow: Nedra, 160 p.

Manual of Instructions for the Survey of Canada Lands. (1996). *Natural Resources Canada Geomatics Canada, 3(1),* 445 p.

National Instrument 43-101 – Standards of Disclosure for Mineral Projects. (2000). *Ontario Securities Commission Bulletin 7815*, 134-142.

Neumyvakin, Yu. K. (1976). *Substantiation of the Accuracy of Topographic Surveys for Design*. Moscow: Nedra, 159 p.

Singer, D. A. & Menzie, W. D. (2010). *Quantitative Mineral Resource Assessments.* Oxford: Oxford University Press, 219 p.

Song, J., Rubert, P., Zheng, A., & Vorburger, T. V. (2008). Topography Measurements for Determining the Decay Factors in Surface Replication. *Measurement Science and Technology, 19*(8), 084005.

Tomlin, C. D. (2013). *GIS and Cartographic Modeling*, London: Esri Press, 324 p.

Triebel, R., Pfaff, P. & Burgard, W. (2006). Multi-level Surface Maps for Outdoor Terrain Mapping and Loop Closing. In 2006 IEEE/RSJ *International Conference on Intelligent Robots and Systems,* 2276-2282.

U.S. Department of the Interior. (2009). *Manual of Surveying Instructions: For the Survey of the Public Lands of the United States.* Bureau of Land Management. Denver, CO: Government Printing Office, 175 p.

Vallée, M. & Sinclair, A. J. (1998). Quality Assurance, Continuous Quality Improvement and Standards in Mineral Resource Estimation. *Exploration and Mining Geology, 7,* 11-22.

Viduyev, N. G. & Polischuk, Yu. V. (1973). Vertical Interval and Topographic Map Scale. *Engineering Geodesy, 14,* 131-137.

Vilesov, G. I. (1973). *Deposit Geometrizing Techniques.* Moscow: Nedra, 184 p.